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Scattering Patterns  
and Statistics of a Long Wire

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## Lincoln Laboratory

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
LINCOLN LABORATORY

SCATTERING PATTERNS AND STATISTICS  
OF A LONG WIRE

*S. HONG*

*Group 41*

TECHNICAL NOTE 1967-55

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## ABSTRACT

The backscattering cross section and pattern of wires 2 to 18.5 wavelengths long were theoretically computed and plotted. Assuming a spherically random orientation of the wire, we present the average cross sections and probability distributions for various lengths and radii of the wire.

Accepted for the Air Force  
Franklin C. Hudson  
Chief, Lincoln Laboratory Office

## Introduction

The problem of the scattering of electromagnetic waves from a wire has received considerable attention. Most of the previous works<sup>1,2</sup> have a drawback in the sense that their solutions are limited to wires not much longer than one wavelength. Recently, a high frequency asymptotic solution for scattering by a wire has been published by Ufimtsev<sup>3</sup>, and his solution is particularly suitable for computation of scattering by wires long compared to the wavelength.

In order to gain some quantitative understanding of the scattering properties and statistics of a long wire, numerical analysis based on Ufimtsev's solution is carried out. In this report the results of computations are summarized, in particular for lengths of wire from 2 to 18.5 wavelengths.

## Basic Theory

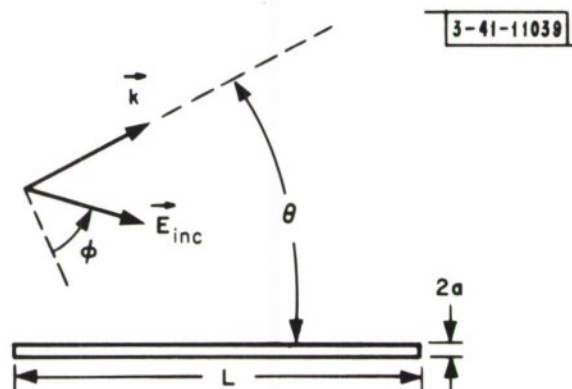


Fig. 1. Geometry of wire.

Consider a plane electromagnetic wave incident upon a perfectly conducting wire of length  $L$  and radius  $a$  at angle  $\theta$ . The angle between the wire and the electric vector is  $\varphi$ . The physical picture of formation of the scattered fields is explained in the following way: when the incident wave hits either end of the wire, the incident wave is scattered, and at the same time a traveling wave is launched along the wire. When this traveling wave reaches the other end of the wire, a portion of its energy is diffracted and reaches the receiver, and the remainder of the traveling wave is reflected and forms a secondary traveling wave. Such phenomena continue with successively smaller amplitudes for the higher order traveling waves. Summation of contributions by various diffracted waves yields the desired solution for back scattering by the wire.

The analytical solution based on this physical picture was obtained by Ufimtsev<sup>3</sup> and is given in the Appendix. The solution is valid when  $\frac{a}{L} \ll 1$  and  $ka \leq 0.2$ . If the wire is not of circular cross section, the solution and results given in this note are still valid if we use an equivalent radius<sup>4</sup>. For a strip the equivalent radius is a quarter of width of the strip, and for a cylinder with elliptic cross section the equivalent radius is the average of the semi-major and semi-minor axes.

### Numerical Results for the Scattering Cross Section

Using the theoretical solution given in the Appendix, the scattering cross sections for various lengths and radii of the wire are computed. Fig. 2 presents the broadside cross section ( $\theta = \pi/2, \varphi = 0$ ) as a function

of length for three values of  $a/\lambda$  (0.01, 0.001, and 0.0001). Note that resonances occur near  $L/\lambda = n + 1/2$  ( $n = 0, 1, 2, \dots$ ). These are due to the fact that the phases of all traveling waves are the same at these lengths and they add up coherently. However, the resonant peaks diminish as the length and/or the radius increases.

Fig. 3 to Fig. 8 presents scattering patterns ( $\varphi = 0$ ) for  $L/\lambda = 2, 3, 5, 8, 12$  and  $18$ . Near broadside, the shape of the angular dependence can be approximated by the function  $\left[ \frac{\sin(kL \cos \theta)}{kL \cos \theta} \right]^2 \sin^2 \theta$  as predicted by Chu's approximate solution<sup>1</sup> for a long wire. However, Chu's solution fails to predict large backscattering returns (so-called endfire lobes) at oblique incidence. It is interesting to note that the maximum cross section does not always occur at broadside. This is particularly true when the wire is very thin. The scattering cross section near broadside depends logarithmically upon the ratio  $a/\lambda$ , but the cross section for endfire lobes is almost independent of the thickness of the wire. Whenever the scattering cross section and pattern for  $a/\lambda = 10^{-2}$  and  $10^{-3}$  are the same only those for  $a/\lambda = 10^{-2}$  are explicitly shown in the figures.

### Statistics

When a wire is randomly oriented (e. g. in a chaff cloud), the statistical properties of the cross section are of interest. For a spherically random orientation, the average cross section,  $\bar{\sigma}$ , is defined by the following equation:

$$\bar{\sigma} = \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin \theta \cdot \sigma(\theta, \varphi) \quad (1)$$

Fig. 9 presents computed values of average cross sections versus length for three values of the radius. As length increases,  $\frac{\bar{\sigma}}{\lambda^2}$  does not increase as rapidly as the broadside cross section shown in Fig. 2. Note that the resonances of the average cross section occur at every half wavelength.

Another interesting statistical property of a randomly oriented wire is the probability distribution  $P(\sigma > \sigma_i)$ . The procedure of computation is as follows: the range of aspect angle ( $\theta$ ) from  $0^\circ$  to  $90^\circ$  are divided such that a sufficient number of sample points are taken with uniform increments of  $\Delta(\cos \theta)$ . The array of the computed scattering cross sections at various aspect angles are multiplied by a second array of eleven values of  $\cos^4 \varphi$  where the range of  $\varphi$  is between  $0^\circ$  and  $90^\circ$  at an interval of  $9^\circ$ . The resulting matrix represents a sufficient number of sample values of the cross sections for a spherically random orientation of the wire. Each sample value normalized with respect to the broadside cross section is then called the XX array in the computer program. This array is ordered from the lowest value to the highest. Finally, the probability distribution  $P(\sigma > \sigma_i)$  at each  $X_i [= \sigma_i / \sigma(\theta = \pi/2, \varphi = 0)]$  is found by computing the percentage of samples whose normalized cross sections are greater than the given  $X_i$ .



Fig. 10 presents probability distributions for  $L/\lambda = 2, 3, 5, 8, 12$  and 18 for  $a/\lambda = 10^{-3}$ . The probability distribution for a half-wave dipole ( $L/\lambda = 0.5$ ) was theoretically obtained by Borison<sup>2</sup>, and the result is also shown in Fig. 10. To show the dependence of  $P(\sigma > \sigma_i)$  on thickness of wire, probability distributions for  $a/\lambda = 10^{-2}$  and  $10^{-3}$  are shown in Fig. 11 (for  $L/\lambda = 2$ ) and Fig. 12 (for  $L/\lambda = 3$ ). Since the  $P(\sigma > \sigma_i)$  for  $a/\lambda = 10^{-4}$  is almost identical to that for  $a/\lambda = 10^{-3}$ , it is not included in the figures. The probability distribution depends weakly on the radius of the wire, because the endfire cross sections are almost independent of the radius of the wire as shown in Fig. 3 to Fig. 8.

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3. P. Ya. Ufimtsev, "Diffraction of Plane Electromagnetic Waves by a Thin Cylindrical Conductor," Radio Engin. and Electron Phys. No. 2, pp. 241-249 (1962).
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# APPENDIX

## UFIMTSEV'S SOLUTION

Computation for scattering by a wire shown in Fig. 1 is based on Ufimtsev's solution. It is given by the following equations:

### I. Case for $\theta \neq \pi/2$

$$\sigma(\theta, \varphi) / \lambda^2 = \frac{4 \cos^4 \varphi \cdot |S(\theta)|^2}{\pi \sin^2 \theta \cdot \sin^2(2\theta) \left| \ln \left( \frac{2i}{\gamma ka \sin \theta} \right) \right|^4} \quad (\text{A-1})$$

where

$$\begin{aligned} S(\theta) = & - \sin^4 \left( \frac{\theta}{2} \right) \cdot \ln \left[ \frac{i}{\gamma ka \sin^2(\theta/2)} \right] \\ & + e^{ikL2 \cos \theta} \cdot \cos^4 \left( \frac{\theta}{2} \right) \cdot \ln \left[ \frac{i}{\gamma ka \cos^2(\frac{\theta}{2})} \right] \\ & + e^{ikL(1 + \cos \theta)} \cdot 2 \cdot \left\{ \sin^4 \left( \frac{\theta}{2} \right) \cdot \Psi_- \cdot \ln \left[ \frac{i}{\gamma ka \sin(\frac{\theta}{2})} \right] \right. \\ & \quad \left. - \cos^4 \left( \frac{\theta}{2} \right) \cdot \Psi_+ \cdot \ln \left[ \frac{i}{\gamma ka \cos(\frac{\theta}{2})} \right] \right\} \end{aligned}$$

$$\begin{aligned}
& + \frac{\cos \theta}{D} \cdot \ln \left( \frac{i}{\gamma ka} \right) \cdot [ e^{ikL^2} (\Psi_+)^2 + e^{ikL^2(1+\cos \theta)} (\Psi_-)^2 \\
& - 2 e^{ikL(3+\cos \theta)} \cdot \Psi_- \cdot \Psi \cdot \Psi_+ ]
\end{aligned} \tag{A-2}$$

$$\gamma = 1.781$$

$$\Psi = \frac{i\pi - 2 \ln(\gamma ka)}{\ln \left( \frac{i2kL}{\gamma k^2 a^2} \right) - E(2kL) e^{-i2kL}} \tag{A-3}$$

$$\Psi_{\pm} = \frac{i\pi - \ln(\gamma^2 q_{\pm})}{\ln \left( \frac{i2kL}{\gamma k^2 a^2} \right) - E \left( \frac{2kL q_{\pm}}{k^2 a^2} \right) \cdot e^{-i2q_{\pm} \frac{kL}{k^2 a^2}}} \tag{A-4}$$

$$q_{\pm} = \frac{(ka)^2}{2} (1 \mp \cos \theta) \tag{A-5}$$

$$E(y) = \int_{\infty}^y \frac{\cos t}{t} dt + i \int_0^y \frac{\sin t}{t} dt - i \pi/2 \tag{A-6}$$

and

$$D = 1 - \Psi^2 \cdot e^{i2kL} \tag{A-7}$$



II. Case for  $\theta = \pi/2$

$$\sigma(\theta = \pi/2, \varphi) / \lambda^2 = \frac{\cos^4 \varphi}{\pi} \cdot |\tilde{S}|^2 \quad (A-8)$$

where

$$\begin{aligned} \tilde{S} = & \frac{ikL}{2A} - ikL \frac{(\bar{\Psi})^2 \cdot E(2kL)}{2A^2} + \frac{A - 1/2}{A^2} \\ & + \frac{2}{A^2} \left[ \frac{\bar{\Psi}}{4} - \ln\left(\frac{i\sqrt{2}}{\gamma ka}\right) \right] \cdot \bar{\Psi} \cdot e^{ikL} \\ & + 2 \frac{(\bar{\Psi})^2}{D A^2} \cdot \ln\left(\frac{i}{\gamma ka}\right) \cdot [e^{i2kL} - \Psi e^{i3kL}] \end{aligned} \quad (A-9)$$

$$A = \ln\left(\frac{2i}{\gamma ka}\right) \quad (A-10)$$

and

$$\bar{\Psi} = \Psi_{\pm} (\theta = \pi/2) \quad (A-11)$$

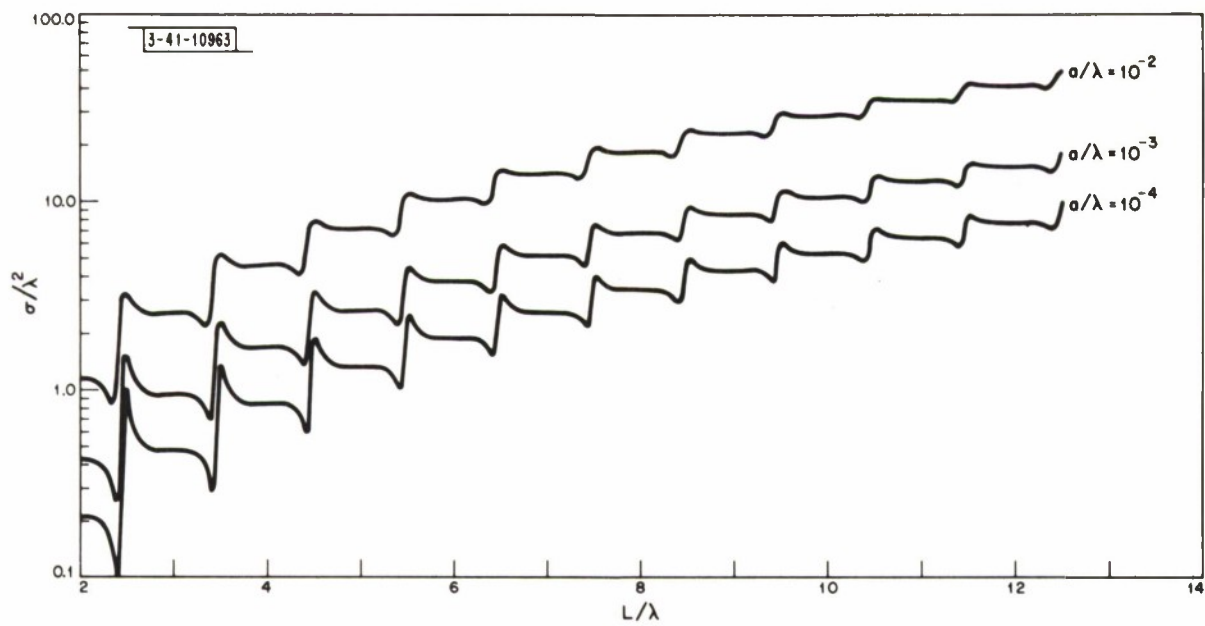


Fig. 2. Broadside cross sections.



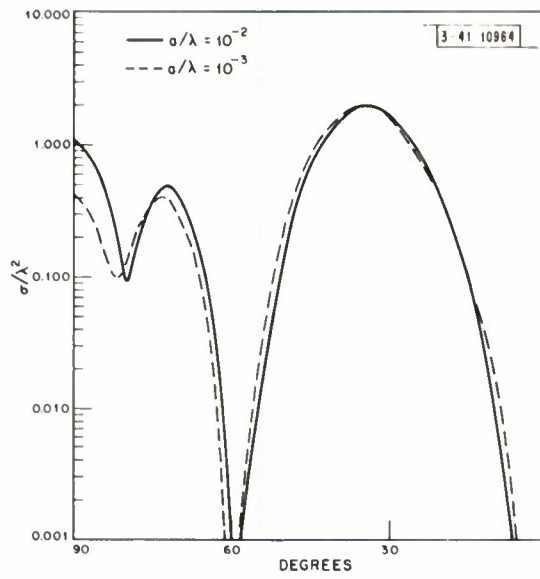


Fig. 3. Pattern for  $L/\lambda = 2$ .

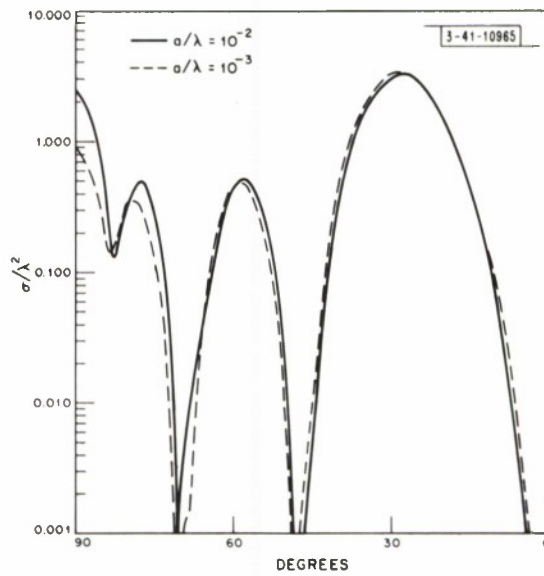


Fig. 4. Pattern for  $L/\lambda = 4$ .

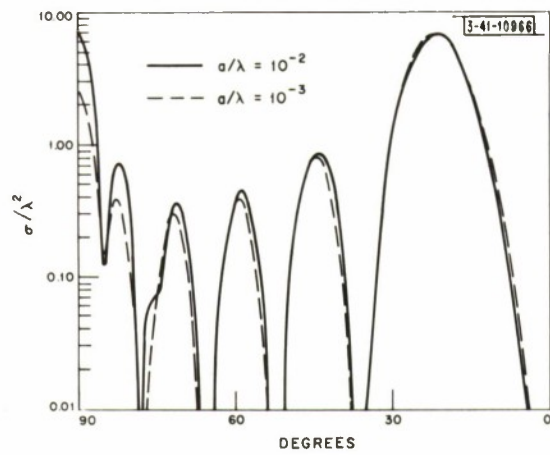


Fig. 5. Pattern for  $L/\lambda = 5$ .

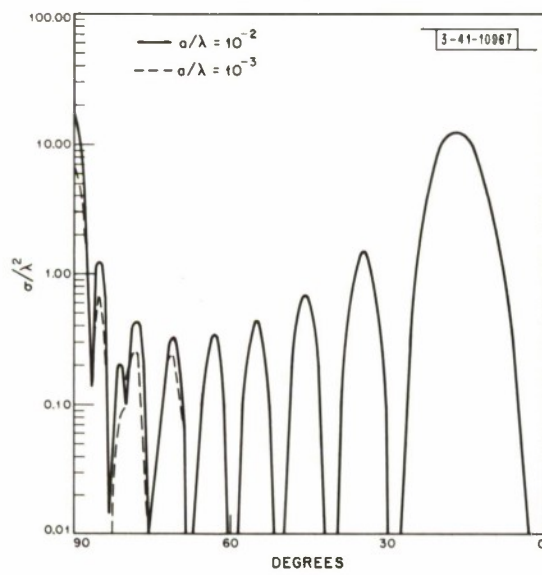


Fig. 6. Pattern for  $L/\lambda = 8$ .



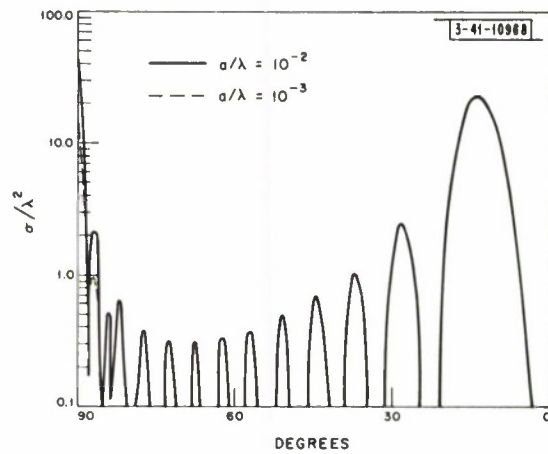


Fig. 7. Pattern for  $L/\lambda = 12$ .

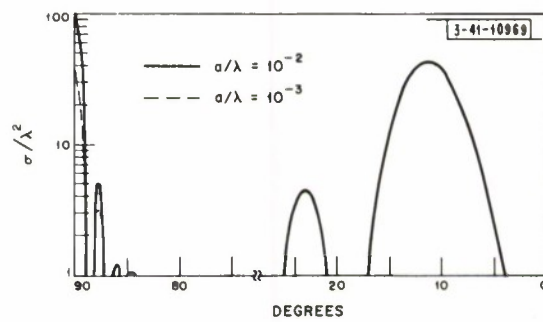


Fig. 8. Pattern for  $L/\lambda = 18$ .

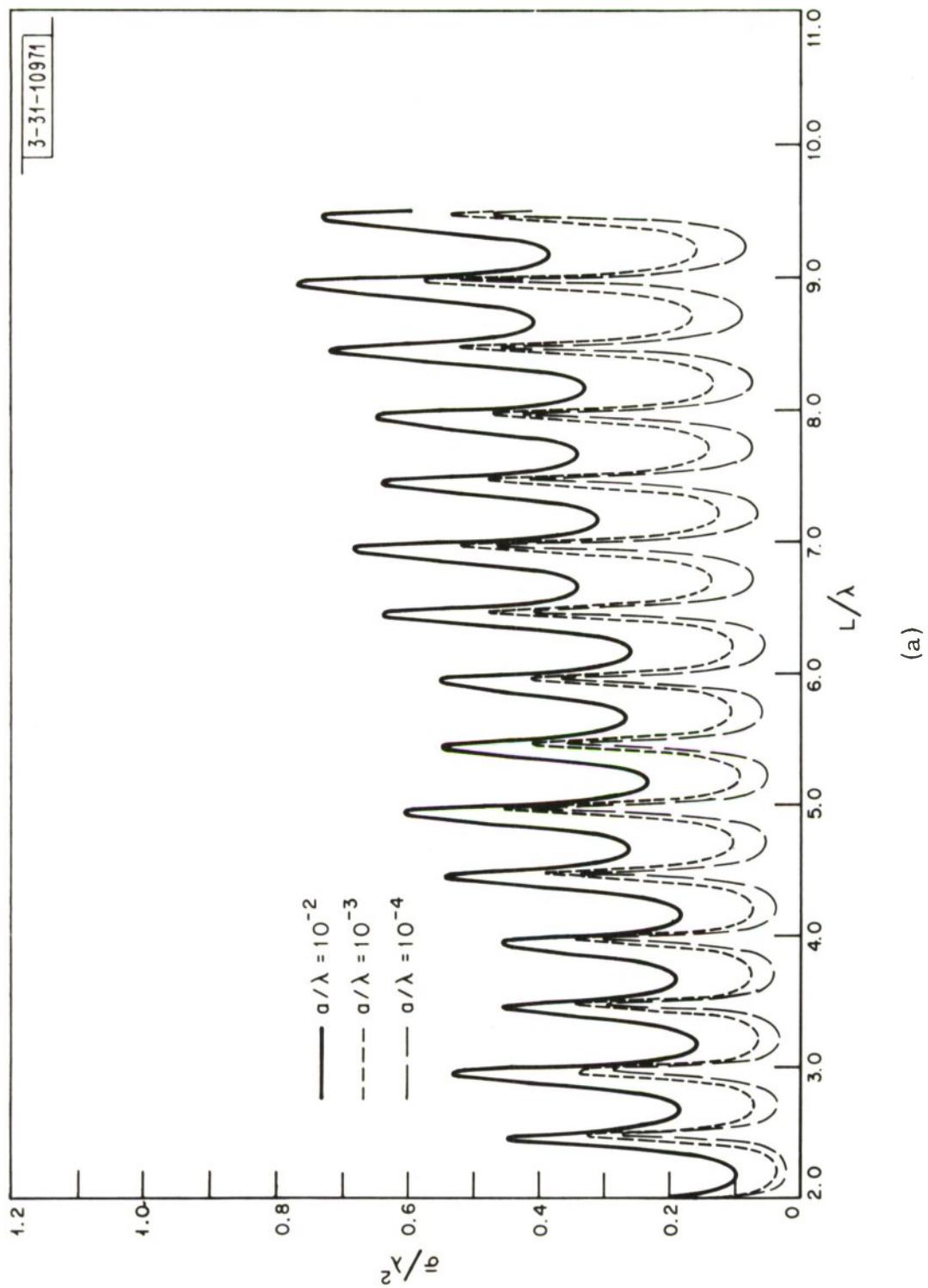


Fig. 9. Average cross section.

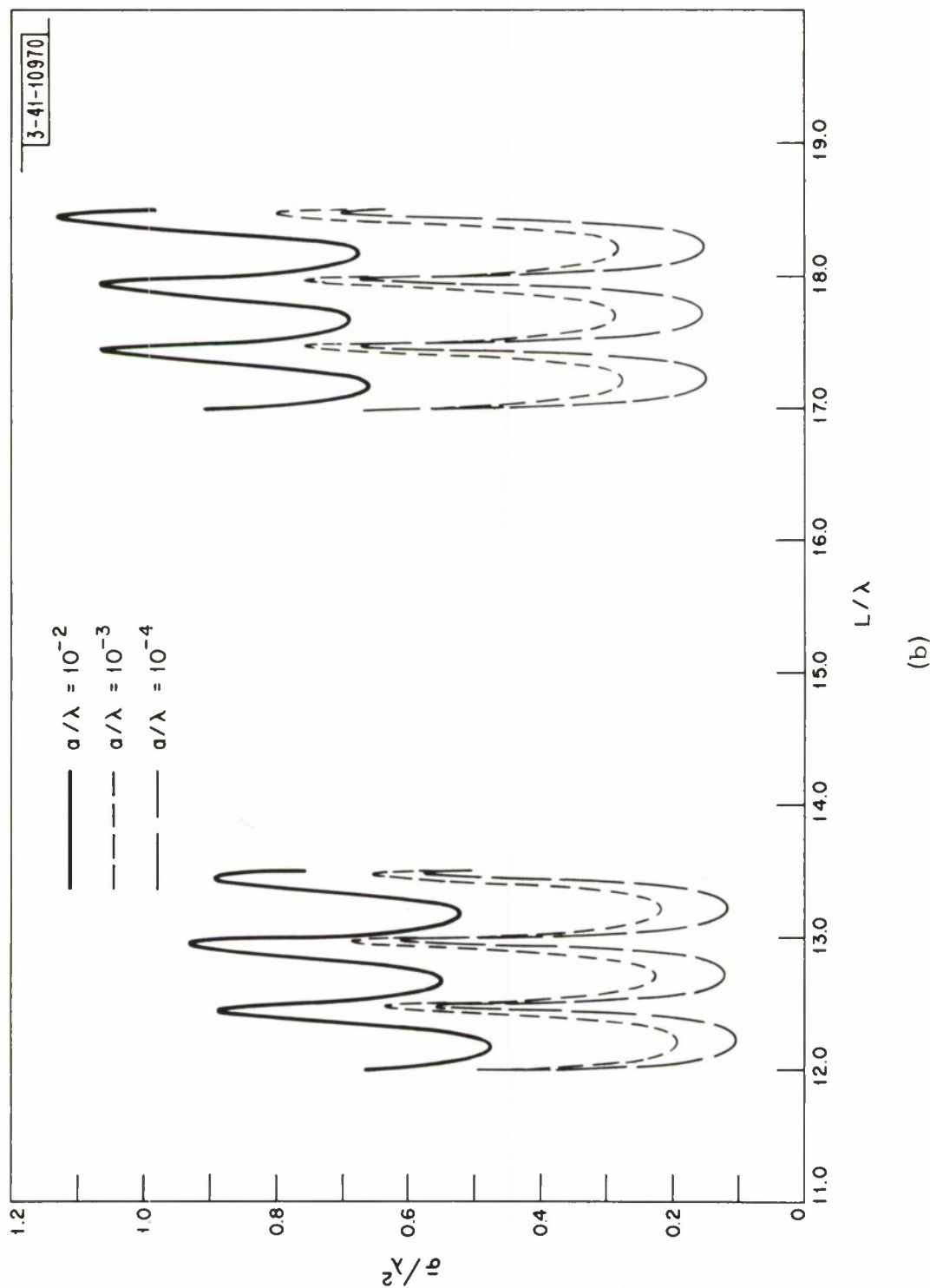


Fig. 9. Continued.



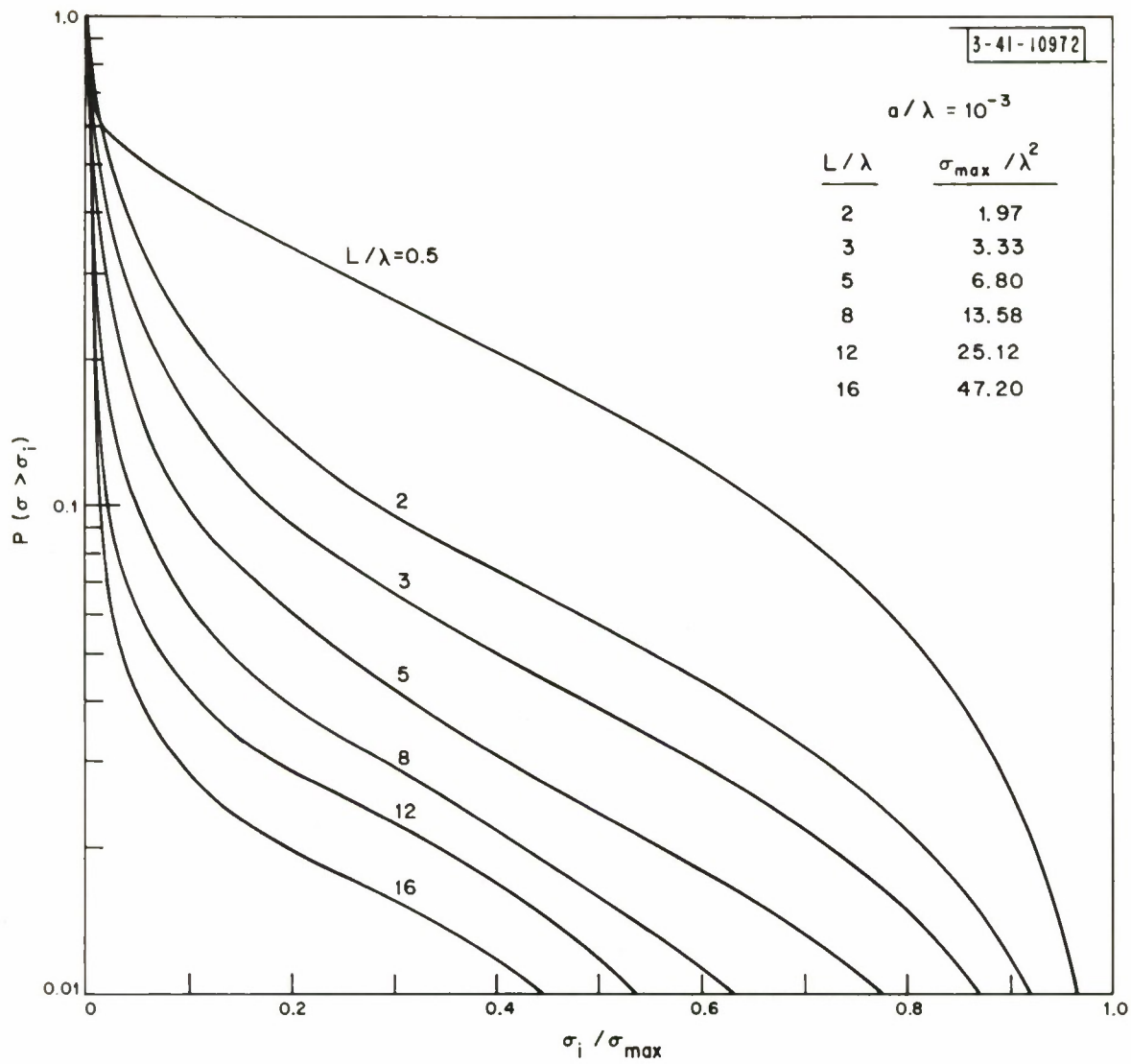


Fig. 10. Probability distributions vs. length.

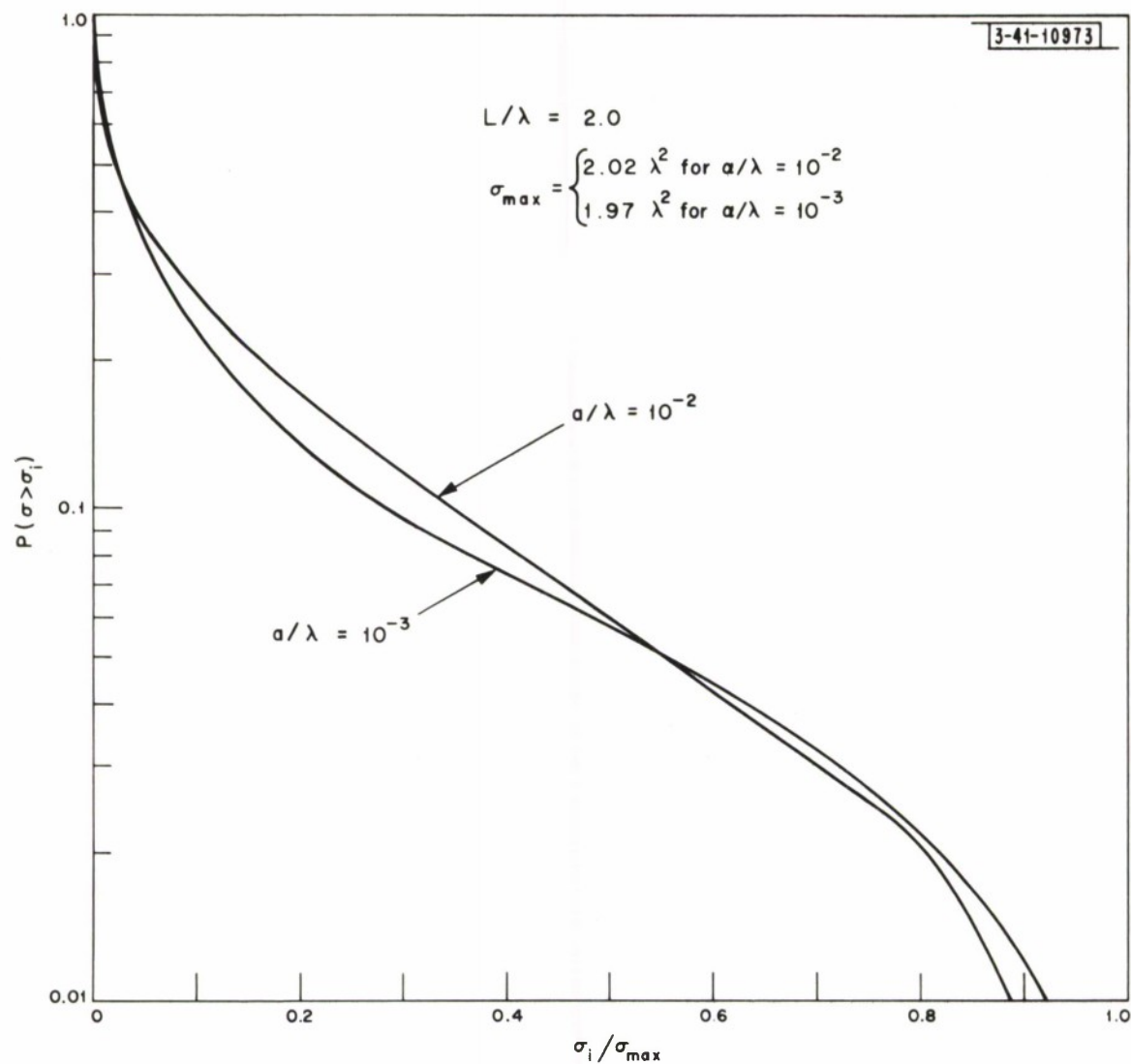


Fig. 11. Probability distributions for two radii and  $L/\lambda = 2$ .

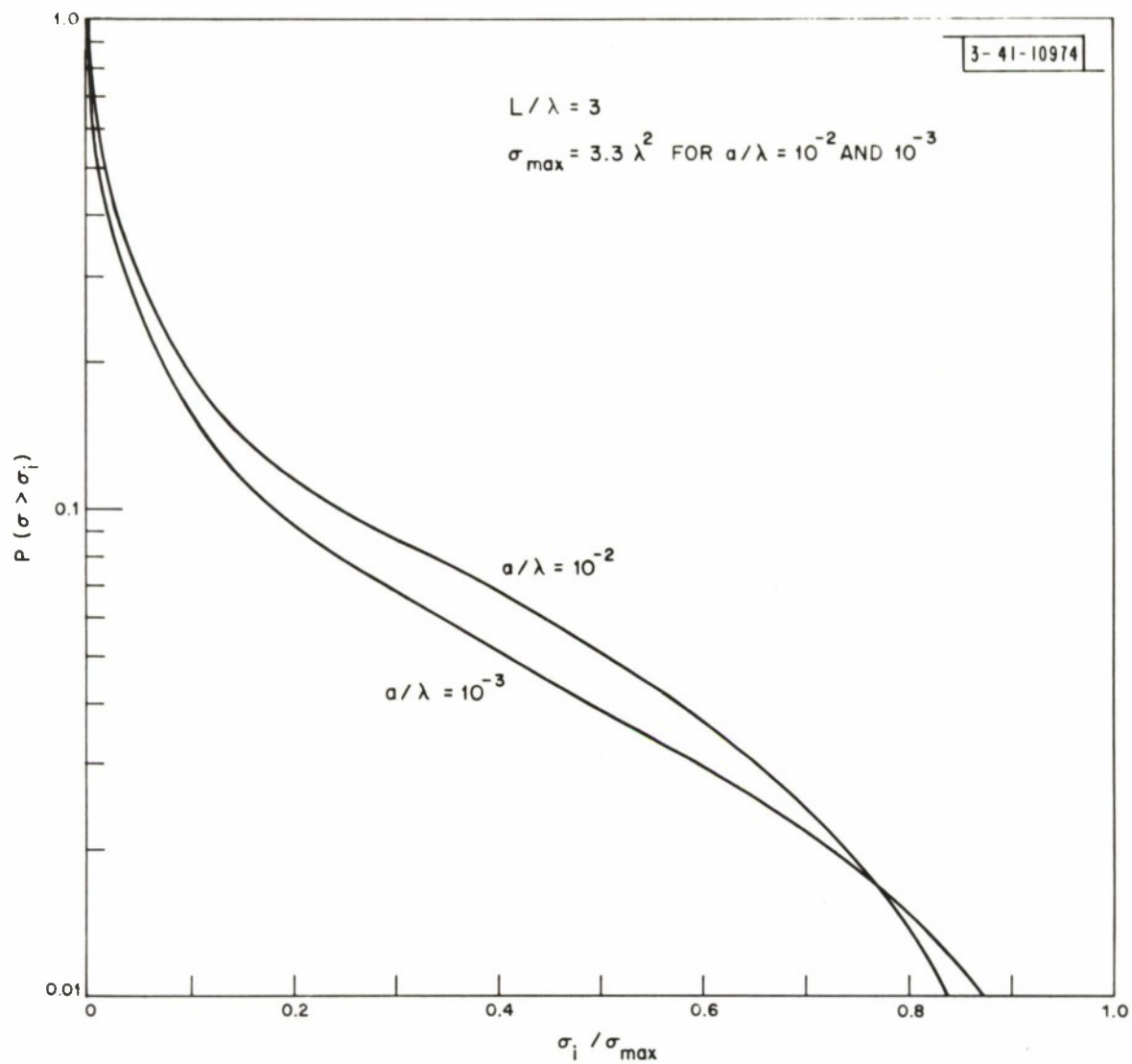


Fig. 12. Probability distributions for two radii and  $L/\lambda = 3$ .



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